Non-traditional Methodology of Instruction Introducing a Primary Historical Source in an Undergraduate Course in Discrete Mathematics

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Introduction

This paper presents a preliminary report concerning a non traditional methodology of instruction which introduces a primary historical source in an undergraduate course in discrete mathematics. The methodology provides an alternative to traditional methods of instruction, that although thorough and mathematically precise, present the course material as a fast-paced news reel of facts and formulae, often memorized by the students, while the text itself offers only passing mention of the motivating problems and original work which eventually found resolution in modern mathematical concepts. The methodology presented in this paper allows the students to hone their verbal and deductive skills through reading, answering key questions, and studying the original works of great minds in history. The students are given the opportunity to react to the original source in much the same way as the contemporaries of the historical masterpiece, explore the development of key and ground-breaking ideas, and rediscover the conceptual roots common to discrete mathematics. The particular course described in this paper is a sophomore level discrete mathematics course titled Introduction to Finite Mathematics in which students learn the basics of logic, naive set theory and proof. A particular form of proof, mathematical induction, is one key tool students should take away from the course. The particular historical source is Blaise Pascal's 1652 *Treatise on the Arithmetical Triangle*. This source by Pascal covers many important notions in mathematics, and it is also the first known work where induction is stated so clearly.

Why use history?

Before discussing the project based on Pascal's work or the course in which the project was used, a short list of some benefits that knowledge of the history of mathematics can offer students (and teachers) is presented. First, this pedagogical approach presents a more humanistic view of mathematics and science in general. The results, techniques developed, and methods used are investigated by students. This provides not only a glimpse into the genesis of new ideas but also the notion that these ideas did not just "pop out of thin air" like a god-given ultimate truth. One can gain insight into the efforts of persons motivated by deep interest and passion, as well as a view of the incomplete and sometimes fallible nature of discovery, which is inherent to science and mathematics. Second, such an approach can give a chronological rendition of the results rather than clumping results from across the ages into a single concept. This "clumping" often does not allow an understanding of why new techniques were developed. Neither does it expose how past fashions influence present fashions nor the deep connections along time of the different movements of the mathematical symphony. Third, such pedagogical methods provide a dynamical vision of the evolution of mathematics; this is important as it seems that many students believe mathematics is "all figured out" and the evolutionary road of mathematics has reached a dead end. This cannot be further from the truth; student exposure to the primordial creativity around particular subjects helps students investigate and research concepts in the infant stages of the theory. This also conveys that research is about new questions, not simply answers to old questions, and helps students get a feel of where questions are generated, thus giving a perspective on the evolutionary progress of mathematics. One might go on giving more reasons about the enriching power of history in mathematical and scientific classroom settings, but the focus of discussion is now directed at an undergraduate mathematics course at NMSU.

Introduction to Finite Mathematics

This section discusses the student population required to take Introduction to Finite Mathematics and the core material of the course. There are two groups of students who take Introduction to Finite Mathematics. The first is those students majoring in mathematics. This is quite often the first time many mathematics majors will encounter a formal course in proof technique. The second is those students that are secondary education majors desiring to teach mathematics. Goals for these students include developing clear and precise methods of communicating mathematics that will assist their future teaching. Previously computer science majors also made up a significant percent of the population of Introduction to Finite Mathematics. Recently at NMSU a new course has been offered for computer science majors that covers similar subject material but is aimed more at how ideas may be implemented in the world of computing.

In the course, first introduced is logic, both classical propositional logic and predicate calculus. The treatment of classical propositional logic is rather quick and is presented via the idea of truth tables. Also the treatment of predicate calculus is quick and informal; the point being to give the students enough working knowledge of predicates and quantifiers to be able to understand and prove statements formulated in the language of predicate calculus. This exposure to logic is followed by an investigation of valid argument forms—this prepares students by allowing them to see the underlying structure of the types of arguments they will produce later in the course. The subject of logic is kept to a minimum as it is not the meat of the course.

After the basic tools from logic are presented, students get practice in using these tools in the setting of naive set theory. The naive notion of a set is given and one uses such to formalize the concepts of relations and functions. One particular type of relation is then introduced—an equivalence relation. For those students continuing to take course work in abstract mathematics this idea of an equivalence relation is very important as it appears in many places in many different areas of mathematics. At this point in the course students are writing proofs using primarily two techniques: direct proof and indirect proof. Indirect proof methods include proof by contradiction and proof by contraposition.

Students are then introduced to statements that are very challenging to prove using any of the techniques previously presented in the course. A new technique, proof by mathematical induction, is introduced at this point. This powerful technique has a wide range of applications within all niches of mathematics. In fact this technique is so powerful that it is a main goal of the course to endow the students with knowledge of how and when to use mathematical induction. In fact in recent semesters a project has been introduced whose goal is to explain mathematical induction based on one of Pascal's papers.

The Project

The project used in Introduction to Finite Mathematics in the last two semesters is based on Pascal's 1652 paper titled *Treatise on the Arithmetical Triangle*. The original paper was written in French, but there is a nice English translation that can be found in Great Books of the Western World, Volume 30. Pascal addresses many issues in this document: properties found within the arithmetical triangle, counting combinations, uses for the orders of numbers, determination of division of stakes in an interrupted game of chance, and expanding powers of a binomial. The project implemented in Introduction to Finite Mathematics focuses on three of the subjects—properties found in the arithmetical triangle, counting combinations and expanding binomial powers. Introducing the project is a historical account of Pascal's life and some motivation about why Pascal did any of this work in the first place.

This paper is ground breaking work and with correspondence with Fermat gave birth to what is now called probability theory. It was proposed to Pascal by Chevalier de Méré, a professional gambler: two players are playing a fair game, to continue until one player reaches the winning number. How should the stakes be divided equitably, based on the number of rounds each player has won? The answer involves combinatorial properties inherent in the number in the Arithmetical Triangle. Together Pascal and Fermat developed a theory that would become the theory of probability and *Treatise on the Arithmetical Triangle* played a crucial role.

Pascal presents many, as he calls, consequences (properties) found in every arithmetical triangle. Many of his proofs have the flavor of mathematical induction. In his twelfth consequence, Pascal very clearly states the principle of mathematical induction within the context of a statement he is proving. This is the first known *complete* written statement and use of mathematical induction. The logic behind induction had been used by others prior to Pascal, but Pascal's paper puts the idea in writing. It is by reading Pascal's proofs that students are exposed to mathematical induction.

The project introduces notations that are common to present day mathematics, but were unavailable to Pascal. Some such tools are index notation and summation notation. Students are asked to read and rewrite many of Pascal's proofs in these new notations. The reason is two sided; first, this gets students to use present day notation to express what Pascal is saying, and second, it allows them to get their hand dirty with Pascal's proofs. Many of Pascal's proofs are well chosen generalizable examples. Students are asked if such proofs are satisfactory and why they believe as they do. Most feel that Pascal did not address the general case though. So the students are asked to improve on these proofs by making them apply for any case, not just the specific one Pascal point out. This is done using present day notations not available to Pascal.

After gaining experience in reading the first sections of *Treatise on the Arithmetical Triangle* and writing proofs of consequences found within the arithmetical triangle, students are then directed to a later section of the paper. This section deals with combination numbers—the numbers that arise as the number of ways one can choose some number objects from a given collection. Pascal gives a nice discussion, that agrees with today's notion, of what it means to choose some number of objects from a given collection. This leads nicely into an investigation of expanding binomials.

After such, the project focuses on congruence relations and prime numbers. The goal of this section is to have the students conjecture Fermat's little theorem and prove it using mathematical induction. This basically involves the students making a table of remainders of a^n upon division by n and then looking for patterns. An optional final part of the project ties together the RSA cryptosystem and Fermat's little theorem.

Results

The project seems to be very successful. Most students grasp the concept of mathematical induction before the present day formal definition, which does not give an intuitive feel for why an inductive argument holds, is presented. The project is run in an interactive setting. By this it is meant that there are many discussions based on Pascal's work in the classroom. During these discussions students share their point of view. Some class days students work in groups; this has many benefits. Some benefits include learning to communicate mathematics verbally with others who are at the same level of mathematical development, students tutoring each other, and bringing to the classroom a discussion based learning experience (which is difficult to pull off successfully in mathematics courses). Most students not only enjoy the class discussions but are active participants. This enjoyment leads to a deeper appreciation of the subject and a greater camaraderie amongst the students.

Pascal's use of mathematical induction is presented in an easy to follow and straight forward manner; even though he never spells out why induction works in general, he gives a wonderful description of how it applies to certain situations with which he is dealing. The students really enjoy Pascal's writing. By the end of the project the students are proving many interesting results using mathematical induction.