Algebraic difficulties in physics

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Abstract

It has been well documented that, when given a written problem statement involving a linear relationship between two quantities, many university students place the constant of proportionality on the wrong side of the equation. Through various survey questions and interviews, we have investigated the role of three possible contributing factors to this difficulty: (1) the role of variable choice, (2) the role of sentence structure, and (3) the role of context familiarity. We discuss each of these factors in turn, and give supporting data from our surveys.

Introduction

The following is a typical examination question from an introductory physics class:

An astronaut finds that she weighs 3 times as much on planet X as she weighs on earth. The radius of planet X is two times the radius of earth. In terms of the mass of the earth, what is the mass of planet X?

This question is intended to test whether the student understands the relationship between weight and mass, and whether the student understands the influence of a planet's radius and mass on the gravitational force that it exerts on objects on the planet's surface. (The weight of an object is given by Newton's law of universal gravitation, $W = G\frac{M_A M_P}{R^2}$, where M_P is the mass of the planet, M_A is the mass of the astronaut, and R is the distance between the centers of mass of these objects.) We have found that only about 10% of 125 students in a calculus-based introductory physics class at New Mexico State University are able to answer this and similar questions correctly. Analysis of student solutions to this problem indicate that, while there is indeed evidence of difficulty with underlying physics ideas (i.e., the relationship between mass and weight), many of the errors in student solutions are algebra errors. Specifically, about 11% of students in a calculus-based class represented the relationship between weight on earth and weight on planet X as $W_E = 3W_X$, instead of $W_X = 3W_E$ and 26% of the students wrote $R_E = 2R_X$, instead of $R_X = 2R_E$. While there were additional algebra errors made in using the universal gravitation equation, we note here that this particular interpretation error alone was responsible for the inability of over 30% of students to answer this question correctly.

As a prelude to developing curriculum to attempt to address this algebraic difficulty, we decided to further characterize its nature. Are students more likely to obtain a correct expression if the variable names are given to them as part of the problem statement? Are students more successful if the question is worded in such a way so that it contains the verb 'is'? Finally, are students more likely to answer this question correctly if they know from the context that one quantity is likely to be larger than the other? In this paper, we give preliminary results based on a pretest and interviews (see Appendix A) given to physics and algebra students containing questions designed to investigate the influence of each of these three factors.

Previous Related Research

The difficulty students have in writing an expression that relates one quantity to another has been studied by many people over the last twenty or so years. John Clement et al {1} investigated the types and source of errors in translation, in particular with reference to a problem they called 'the students-and-professors problem':

> *Write an equation for the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.*

They found that it was common for students to write $6S = P$ or an equivalent equation when translating the above statement. They identified two distinct sources for this error:

1) *'word order matching' or 'syntactic translation'*

and

2) *'static-comparison'.*

The first source of error, the word order matching or syntactic translation, occurs when students transfer the order of the information from the English statement directly to the algebraic equation. Thus the student interprets 'six times as many students as professors' as *6 times students gives professors*, or $6S = P$. The second source of error, 'static-comparison', occurs when the student misunderstands the role of the letters, or variables, in the mathematical equation: $6S$ is used to represent the larger group. Another way to interpret this error is that the variable S (students) is simply treated as a label or place holder with the number *6* telling how many there are of those in relation to the other quantity labeled by P (professors).

Rosnick and Clement have found {2} that these errors are very persistent. They used questions similar to the students-and-professors problem above to show that tutoring had very little effect on students' understanding of their error in translation. Even given problems where the context would suggest that one quantity should be larger than the other (i.e. as in the students-and-professors problem), students would find ways to rationalize their incorrect mathematical interpretation of the English statement. For example, one student, when asked to translate *'there is three times as much oil as vinegar in a salad dressing',* said her first impulse was to write $3O = V$. She recognized this as incorrect and changed the equation to $3V = O$. But when asked to discuss her solution further, she came out in the end saying there was more vinegar than oil, drawing a picture showing a cup of vinegar that was three times as full as the cup of oil. The authors propose that she had learned some algorithmic way of considering the translation of the English into a mathematical equation, but still had no understanding of what the equation meant, in particular the variables.

Rosnick considered this question further {3}, having asked 152 students to describe what the letters *S* and *P* represent in the equation $S = 6P$. He found that over forty percent of the students were unable to accurately pick 'number of professors' as the answer for what *P* represented. This led us to the conclusion that many students, even those in higher-level calculus and physics courses, do not truly understand the concept of variable. A more recent work by Trigueros and Ursini {4} delves into this question at length. They identify at least three uses for variables in elementary algebra courses: 1) specific unknown, 2) general number, and 3) functional relationships. They point out that:

> *University students cannot differentiate between the different uses of variable. While they are capable of recognizing the role played by the variable in very simple expressions and problems, any small increase in complexity provokes inadequate generalizations…Students' understanding of the concept of variable lacks the flexibility that is expected at this educational level.*

Context for Research

We present here data from questionnaires given to students in an introductory calculus-based physics course and an introductory algebra course at New Mexico State University. Students enrolled in introductory calculus-based physics course are primarily engineering majors. Thus, we assume a greater level of sophistication and algebraic understanding in these students since they have often already completed at least one semester of the higher-level calculus course or are taking this course concurrently. Most of the students enrolled in the algebra course have had difficulty with algebra in the past and we expect that they would have more difficulty with the questions we asked.

Contributing Factors to Algebra Errors

The questionnaire we designed was administered at the beginning of the Fall 2003 semester. The purpose of the questionnaires was to determine whether the students can correctly translate an English statement similar to the students-and-professors problem into a mathematical equation. We explored whether the label used to denote the missing quantities, the wording of the problem statement, and the context had any effect on the outcome. A complete questionnaire is shown in the Appendix. We also conducted interviews with the algebra students as part of our investigation of these errors.

Role of variable definition

We found that giving the students a particular letter to use to represent the quantities described had no measurable effect on their ability to translate the English sentence correctly. That is, we included a statement about the definition of the variables to use as is shown in the original students-and-professors problem above (*Use S for the number of students and P for the number* *of professors)*. It *did* help with analyzing results since many students, especially the ones in the algebra course, were not particularly careful about defining their variables.

Role of sentence structure

A relationship between two quantities that are proportional to one another is commonly expressed in one of two ways:

a) *The number of cats is 6 times the number of dogs.*

or

b) *There are 6 times as many cats as dogs.*

Clement et al {1} found that students will often translate an English sentence into mathematics in the order the words are written (syntactic translation). Based on this we expected that almost all students should be able to correctly translate problems using a sentence structure similar to a) above. However, we found that many students (29% in physics and 39% in algebra) made the same 'switching' mistake with *either* sentence structure. We realized when analyzing student responses that the order in which we put the problems on the questionnaire may have had an effect on student performance. The first question students encountered used wording b), which may have influenced the translation of subsequent problems no matter what word order was used. In interviews, we gave students the questions in random order and found that, indeed, when students encountered a problem using wording b) first, they would second-guess how to write the translation of a subsequent problem using wording a).

We did find on the questionnaires and in interviews that students will often translate problems using wording b) in the order the information is given, i.e., the problem above would be translated into the mathematical equation $6C = D$, where C represents the number of cats and D represents the number of dogs (38% of physics students and 46% of algebra students). From the work of Clement et al {1}, Rosnick and Clement {2}), and Rosnick {3}, one possible explanation for this would be that the labels C and D do not actually represent number of cats and number of dogs respectively, but serve as labels or place holders, their 'static comparison'. From interviews, however, we began to suspect that this *source* of the error was actually an *effect* of the syntactic translation (word order) error. This was reinforced by students' reactions to writing a syntactic translation for a problem with a context that should suggest which quantity should be larger, as explained below.

Role of context

When context does not give the students a cue about expected size of the quantities represented by the variables, those who make the switch in their equation are quite happy to use syntactic translation, or word order, (i.e. they will accept $6C = D$ as a correct translation of a problem with wording b) above). However, when they encounter a problem that gives a contextual cue, we have noticed that they then pause as if puzzled by the resulting equation. Some choose to change the order of the relationship and will then correctly write $6D = C$, but are often uncomfortable with that change. The resulting explanation of C or D as a place holder comes as a result of either trying to explain the new equation to themselves, or in wanting to explain why their original equation should be the correct one. They are more willing to change the meaning of the variable than they are to change the way they translate the English statement, even to the point of making a switch in translation when the word order would give them the correct equation.

Conclusion

Although our interview results were similar to those recorded in Rosnick and Clement {2}, we have arrived at a somewhat different conclusion. Though students *do* have difficulty with understanding the role of variables in algebra, as thoroughly attested to by Trigueros and Ursini {4}, we wonder if this difficulty comes in part from a tenacious desire to hold on to an algorithm they have learned for translating English into mathematics. Is the decision to redefine the role of the variable a *result* of the way they have been taught mathematical modeling rather than a separate source of translation errors?

Clement et al mention {1} that students who are most successful in translating the students-and-professors problem correctly are those who use an 'operative translation' approach. In other words, the students understand the role the variables play in the English statement and are willing to fit the contextual relationship of the quantities into a mathematical equation that would make sense in that context rather than following a particular algorithm for translation.

Often students are drilled in the syntactic translation, or word order, approach to problem solving, apparently to their detriment in various ways. When they encounter a problem such as the ones described above, they are often unable to translate them correctly. We believe that this leads to further confusion about the role of variables as they try to justify their translation in light of contextual cues. These results imply that there is a need to address how we teach problem solving and mathematical modeling in *all levels* of algebra instruction, in addition to further investigation of difficulties students have with understanding the role of variables.

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References

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APPENDIX

Algebra Diagnostic Pretest

1. Convert the following statement into mathematical symbols. Do not solve or simplify

   There are five times as many pins as there are snaps in my sewing kit.

2. Convert the following statement into mathematical symbols. Do not solve or simplify.

   The number of pencils in my desk drawer is five times the number of paper clips.

3. Using *h* for the number of 'hephalumps' and *w* for the number of 'woozles', convert the following statement into mathematical symbols. Do not solve or simplify.

   There are three times as many 'hephalumps' as there are 'woozles' in Winnie the Pooh's dream.

4. Using x for the number of crabs and y for the number of starfish, convert the following statement into mathematical symbols. Do not solve or simplify.

   There are three times as many crabs as there are starfish in the aquarium.

5. Convert the following statement into mathematical symbols. Do not solve or simplify.

   There are two times as many feet as there are heads in the picture.

6. Convert the following statement into mathematical symbols. Do not solve or simplify.

   The number of hooves in the picture is two times the number of horns.