

# **Student use of vectors in Introductory mechanics**

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A functional understanding of Newton's second law presupposes an understanding of basic vector operations. In order to find a net force, students must understand vector addition. In order to make sense of the direction of acceleration, students must be able to reason about the change in a velocity vector. We describe initial results from our investigation into student difficulties with vector addition and subtraction in the introductory calculus-based physics course. We have conducted student interviews on use of vmany students still struggle with vectors after all instruction in mechanics. Some common difficulties are described, as are results from some attempts to address these difficulties.

## **Introduction**

During recent years, a great number of physicists have contributed to the development of a fundamental field in scholar life: the teaching and learning of physics. Most of these physicists are willing to investigate about the disparity between what teachers teaches and what student learns in physics courses [1]. These researchers have conducted systematic investigations into student understanding of specific topics in physics. Curriculum has been designed based on this research, tested in the classroom, and modified based on this testing. Through an iterative cycle of research, curriculum development, and assessment, significant progress has been made in improving student performance in the introductory course and beyond [2]. However, there are many topics that still lack the solid research foundation that is necessary for the development of effective curricula. We have identified student use of vectors as one of these topics, and have begun an investigation into student use of vectors in various contexts. Previous research designed to explore student understanding of kinematics has found that in some contexts many students fail to differentiate between position and displacement or between velocity and acceleration [3]. If these students are asked to figure out the direction of the acceleration vector when an object is moving in a circular path, it was found that with constant speed, increasing speed or decreasing speed, many students answer that the acceleration vector has direction toward the center of the circle. It appears that many students mistakenly generalize the result obtained for circular motion at constant speed, or they memorized a rule for acceleration without understanding the relationship between this rule and the definition of acceleration. We would like to determine the degree to which difficulties with kinematics are a result of a failure to understand the basic vector operations of addition and subtraction.

## **Motivation for investigation**

This paper describes some results from an investigation into student understanding of vector operations. The main reason of this research is due to the student conceptual difficulties that instructors have found frequently in introductory physics courses [4]. Vectors are essential in the development of a functional understanding of Newton's second law and kinematics. We have found that many students have difficulties with the use of vectors when solving both conceptual exercises and in traditional problems. The purpose of this investigation is to understand the nature of these difficulties and to establish a basis for the development of curriculum to improve student understanding of vectors.

## Context for investigation

Our investigation has been conducted in introductory physics courses at NMSU, the University of Washington, and the Syracuse University. We have identified difficulties in conceptual understanding of vectors through interviews with students and through analysis of students' written responses to questions asked on examinations. We expect that students should, as a result of instruction, be able to add and subtract vectors and to recognize contexts that require use of vectors rather than scalars. We have found that after traditional instruction many students cannot add and subtract vectors properly.

## Two examples in different contexts

I) Figure 1 shows a question asked on a final examination at NMSU and at Syracuse University in order to determine the degree to which students understand the concept of change in velocity. We expect that after instruction in mechanics, students should be able to subtract the initial velocity vector from the final velocity vector as shown in figure 2, in order to obtain the correct answer (choice c).

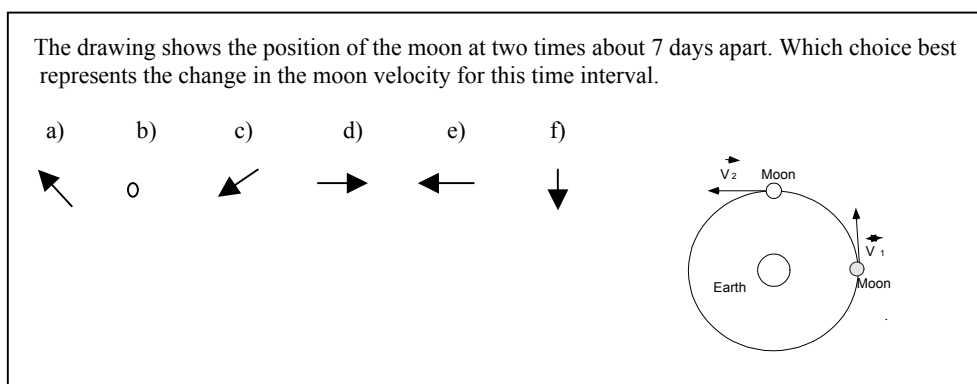


Figure 1. *Moon's question.*

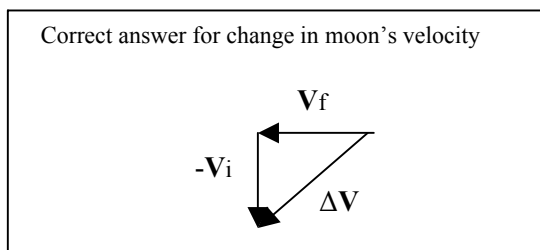


Figure 2. *Construction to obtain correct answer to moon question.*

At NMSU (N=100) only 20% of the students answered the question correctly the question after traditional instruction. At Syracuse University (N=272), 52% answered correctly after instruction that included use of *Tutorial in Introductory Physics* [5], curricular material designed by the Physics Education Group at the University of Washington to reinforce student conceptual understanding. These tutorials include specific exercises using inquiry as a didactic to address many student difficulties identified by physics education researchers, including difficulties related to find the direction of acceleration. (A more detailed discussion of the use of these materials can be found in reference 5). When instruction at NMSU was modified, such that the instructor worked with a great emphasis in conceptual understanding in kinematics and Newton's second law, 90% of students were able to answer correctly (N=75).

II) Figure 3 shows a second question about vectors asked in order to determine the extent to which student difficulties are difficulties with the context of kinematics and dynamics rather than difficulties with the vector operations. This question was asked as an examination problem at NMSU (63 students), at Syracuse University (209 students) and at the University of Washington (82 students).

Shown at right are two vectors, **A** and **B**, each of length 6.  
 Let  $C=A+B$ . Is the magnitude of **C** greater than, less than,  
 or equal to 6 ?

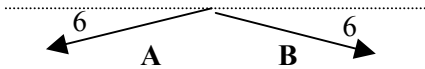


Figure 3. Question asked about addition of vectors.

The correct answer is that the magnitude of vector **C** is less than 6. Figure 4 shows the procedure to find this answer. As shown in table1, we have found that only 51% of the 63 students from NMSU were able to answer correctly. 30% of students were unable to answer correctly because they used Pythagoras. It is interesting to note that 42% of students did not use a diagram to answer the question. About half of these students obtained the correct answer.

We found that 20% of the students add vectors **A** and **B** as scalars to get an answer of 12 for the magnitude of **C**.

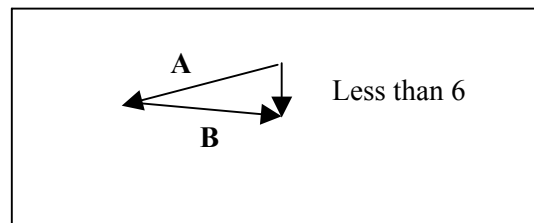


Figure 4. Procedure to find vector **C**

	NMSU N= 63 students	Syracuse University N= 209 students	Univ. of Washington N= 82 students
Less than (Correct)	51 %	79 %	92 %
Greater than	39 %	14 %	4 %
Equal to	10 %	6 %	2 %
No answer	0 %	1 %	2 %

Table 1. Results from vector addition question.

At NMSU 10 % of students changed the direction of both vectors (**A** and **B**) during the addition process, from which 100% found the correct answer with a change in the direction of **C**. We found that all students from 6% who showed an essentially correct but incomplete reasoning got the right magnitude of **C**. Just 70 % from 21% of students who showed a totally wrong reasoning answered the question correctly. These students have a better numerical understanding than a conceptual understanding.

## Observations

One of the most important differences between NMSU and both the Syracuse University and University of Washington, is that at NMSU Tutorials in Introductory Physics is not used in introductory physics courses but is used not only at Syracuse but also at the University of Washington. At NMSU students were students were taught with traditional instruction. Even at the universities with augmented instruction, students performed failed on parts of these questions. It appears that more research in necessary to develop more affective curriculum.

## Subsequent investigations

In the future we are going to continue this research by monitoring the progress of students in introductory physics courses at NMSU. The objective of this investigation will be to learn whether students can obtain a batter conceptual understanding with an increase of a conceptual emphasis in vector addition and subtraction.

## Conclusions

Many students do not develop a meaningful understanding of vector addition and subtraction as a result of traditional instruction. Despite the use of a modified instruction, many students are unable to answer simple questions about vectors.

## References

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- {4} F. Reif and S. Allen, "Interpreting and teaching scientific concepts, a study of acceleration," Berkeley Cognitive Science Report Series, Department of Physics and School of Education, University of California at Berkeley, March 1990.
- {5} L.C McDermott, P.S. Shaffer, and The Physics Education Group at the University of Washington *Tutorials in Introductory Physics*, (Prentice Hall, Upper Saddle River, NJ. 1998). *Tutorials in Introductory Physics* is a two-volume set. One volume contains the tutorial worksheets; the other consists of the tutorial homework.

